$$I_1 = \frac{d\rho}{d(H^2)} A_1.$$

From the equations of motion we have

$$H^2\rho^2 = \frac{2c^2Vm_1}{e_1} \text{ and } \frac{d\rho}{d(H^2)} = \frac{\rho}{H^2} = (\text{constant}) \frac{e_1}{m_1},$$

and therefore

$$I_1 = A_1 (\text{constant}) \frac{e_1}{m_1}.$$

We conclude that the areas of the peaks due to the different kinds of ions should be divided by the corresponding value of m/e in comparing ion abundancies.

[12] $\frac{I^+_{Hg}}{I^+_{CO}} = \frac{p_{Hg} P_{Hg}}{p_{CO} P_{CO}}$ where $\frac{I^+_{Hg}}{I^+_{CO}}$ represents the ratio of the abundance of positive mercury ions to positive carbon monoxide ions as measured, p_{Hg} and p_{CO} represent the partial pressures of mercury and carbon monoxide, respectively, and P_{Hg} and P_{CO} represent the probabilities of ionization by impact between electrons and mercury atoms and between electrons and carbon monoxide molecules, respectively.

[13] L. Loeb, *Kinetic Theory of Gases*, p. 513.

[14] $\frac{I^-_{CO}}{I^-_{Hg}} = \frac{n_{Hg} p_{CO}}{n_{CO} p_{Hg}}$ where $\frac{I^-_{CO}}{I^-_{Hg}}$ represents the ratio of the abundance of carbon monoxide negative ions to mercury negative ions, n_{Hg} and n_{CO} represent the attachment constants for mercury and carbon monoxide, respectively, and where p_{Hg} and p_{CO} are defined as in note 13.

THE PRESSURE OF THE WIND ON LARGE CHIMNEYS

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NATIONAL BUREAU OF STANDARDS, WASHINGTON, D. C.

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Although the circular cylinder is one of the simplest geometrical forms, the phenomena attendant upon the flow of air around it are extremely complicated. It has not as yet been possible to give a complete theory of the motion of the air, but many properties of the flow have been determined by experiment for the range of sizes and speeds available in wind tunnels. The experiments which are here outlined were made in connection with the practical problem of determining the wind pressure exerted on a large chimney in high winds, and give additional information as to the flow around large cylinders.

For a certain range of sizes and speeds, it is found that the total force exerted by the wind is proportional to the area of the projection of the cylinder on a plane normal to the wind and to the velocity pressure (i.e., one-half the product of the density of the air by the square of the wind speed). The ratio of the force to the product of projected area and velocity pressure is called the force coefficient. A more exact analysis shows that the force coefficient is a function of the so-called Reynolds Number, i.e., product of velocity by the diameter of the cylinder divided by the kinematic viscosity of the air. At the highest Reynolds Numbers reached in ordinary wind tunnels, the force coefficient is found to decrease very rapidly with increasing Reynolds Number, reaching a minimum whose value is one-third of the value at smaller Reynolds Numbers. In other words, the wind force on a large cylinder at a given speed is very much less than that which would be obtained by increasing the force on a small cylinder at the same speed in the ratio of the exposed areas, or alternatively, the force on a given cylinder at high speed is much less than that which would be obtained by increasing the force at low speed in the ratio of the squares of the speeds.

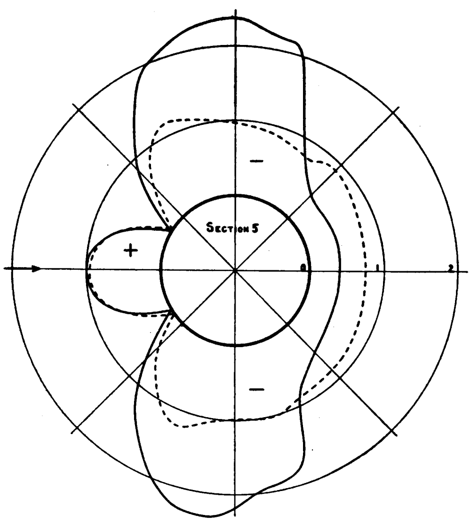

FIGURE 1

Distribution of pressure on 8-inch cylinder. Radial distances from the base circle are proportional to the ratio to the velocity pressure of the difference in pressure between the station and the atmospheric pressure. In the region marked +, the pressure on the cylinder exceeds the atmospheric pressure, while in the region marked —, the pressure on the cylinder is less than the atmospheric pressure. The wind direction is shown by the arrow. The broken line is for a speed of 40 ft./sec., the solid line for 80 ft./sec.

We have measured the changes in the distribution of the wind pressure around the cylinder occurring at this so-called critical region and some of the results are shown in figure 1. The heavy black circle represents the cylinder, with air approaching in the direction shown by the arrow. The changes in pressure from the normal atmospheric pressure which are produced by the wind are plotted radially outward. The region marked plus is a region of increased pressure, the region marked minus a region of decreased pressure. The unit for measuring the pressure is the velocity pressure. The dotted curve refers to a low speed (40 ft./sec.) and the solid curve to a high speed (80 ft./sec.) for a cylinder 8 inches in diameter. The extensive area under reduced pressure is surprising. A little study shows that in spite of the larger reductions in pressure at the sides, the solid curve corresponds to a lower force in the direction of the wind.

Attention is called to the relatively small change in the angle at which the pressure equals the normal atmospheric pressure as compared with changes in the magnitude of the reduction in pressure at the sides or at the rear.

To obtain information on cylinders comparable in size to large chimneys, it is necessary to supplement experiments in wind tunnels by experiments on large cylinders in natural winds. The principal difficulty in such experiments is the determination of the velocity of the wind. It occurred to one of us that it might be possible to secure the wind speed from the

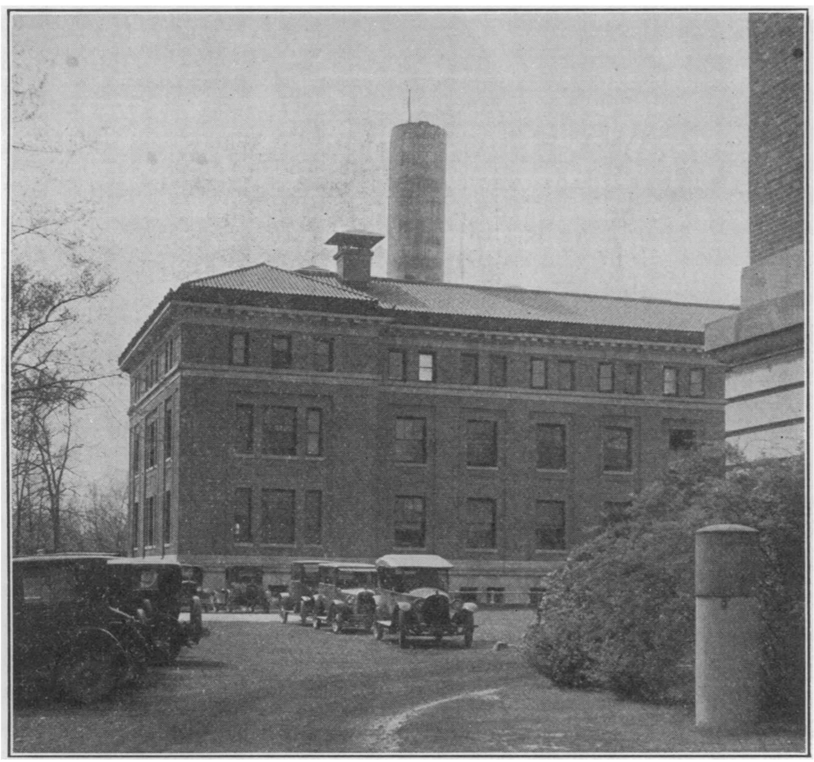

FIGURE 2

Experimental stack, 10 ft. in diameter, 30 ft. high, erected for measurements of wind pressure in natural winds.

pressure distribution diagram by estimating the angle at which the pressure is equal to the normal atmospheric pressure, and determining the maximum increase in pressure above the normal atmospheric pressure, thus obtaining the velocity pressure. It has already been pointed out that this angle is not as variable as other properties of the distribution.

Since the Bureau of Standards was to erect a new power plant chimney on which a more or less permanent installation could be made, it was considered advisable to try the new method on a somewhat smaller scale and under such conditions that the natural wind measurements could be made to stand alone. For this purpose an experimental stack 10 feet in diameter and 30 feet high (as shown in figure 2) was erected on the roof of the West building of the Bureau. Pressures were measured at twenty-four stations around the circumference at a single elevation, about two-thirds of the height from the base of the stack. The wind speed was determined by means of a pitot-static tube mounted on a weather vane about ten feet higher than the top of the stack. The overturning moment was measured by mounting the stack on pressure capsules connected to pressure gauges. The value of the average wind pressure determined from the over-

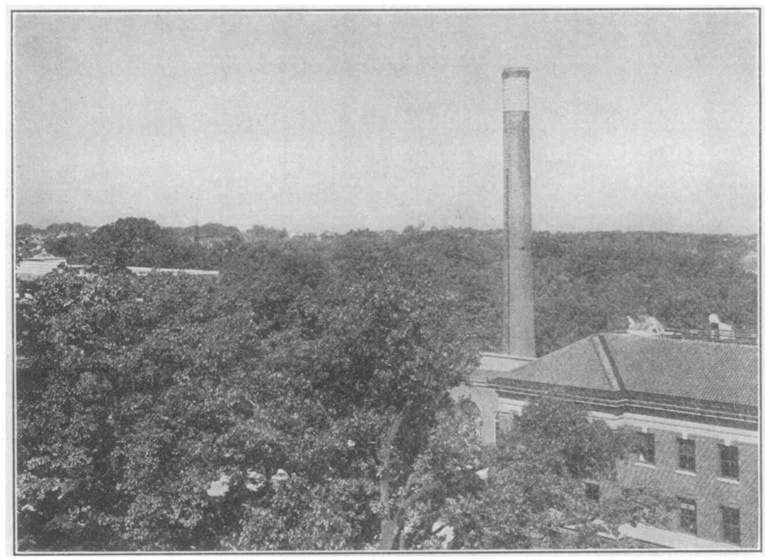

FIGURE 3

The power plant chimney of the Bureau of Standards.

turning moment was somewhat higher than the value determined from the pressures at a single elevation. The method of determining the velocity from the distribution itself was found to be reasonably satisfactory when the various attendant circumstances were considered, especially the small value of the length-diameter ratio of the experimental stack.

On the occasion of the construction of a new power plant at the Bureau of Standards, provision was made for measurements of the distribution of wind pressure on the stack at a single elevation so that some information might be obtained as to the wind pressure on an actual chimney. A view of the stack and its surroundings looking toward the northwest (the direction of the prevailing wind) is shown in figure 3. The dark line running down the stack is the group of 24 pipes leading to the pressure gauge and the holes are located about 2 feet above the top of the pipes. The stack at this point is 11.8 feet in diameter.

Space does not permit any detailed statement or comparison of the results. A full account of the work is given in *Research Paper* **221** appearing in the September, 1930, issue of the Bureau of Standards *Journal of Research.* The general conclusions drawn from the tests may be stated as follows:

1. The wind pressure on a chimney at a given wind speed is a function of the ratio of the height of the chimney to its diameter and possibly also of the roughness of its surface.

2. Experiments on small cylinders cannot be directly used to predict the wind pressure on a full scale chimney because of the large scale-effect.

3. A wind pressure corresponding to 20 lbs. per square foot of projected area at a wind speed of 100 miles per hour is a safe value to use in designing chimneys of which the exposed height does not exceed 10 times the diameter.

4. The pressure may reach large values locally and this may need consideration in the design of thin-walled stacks of large diameter.

5. Further experiments are necessary to obtain satisfactory information as to the variation of wind pressure with the ratio of height to diameter.

PROJECTIVE NORMAL COÖRDINATES

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A projective geometry of paths was formulated first by T. Y. Thomas,[1] and included the definition of a type of projective normal coördinates. Veblen and J. M. Thomas[2] proposed a system of such coördinates as solutions of a certain system of partial differential equations in n dependent variables. In §41 of my Non-Riemannian Geometry[3] I showed that the determination of these coördinates could be reduced to the solution of a differential equation in one dependent variable. It is the purpose of this note to give this equation another form and thence obtain the explicit form of the expressions of general coördinates in terms of projective normal coördinates.

1. Consider an n-dimensional space V_n of coördinates x^i. We take as the basic elements of V_n the *paths*, that is, the integral curves of the system of differential equations.